TRINITY COLLEGE

IN NOMINE DOMINI

YEAR 12
MATHEMATICS
SPECIALIST

Test 3, 2023
Section One: Calculator Free
Integration Techniques and Differential Equations

STUDENT'S NAME: ____

DATE: Wednesday 16th August **TIME:** 25 minutes **MARKS**: 25
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items:

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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Question 1 **(8 marks)**

(a) $\int x^2 \sqrt{3 + x^3}\, dx$ (2 marks)

(b) $\int x \sqrt{x + 1}\, dx$ Use the substitution $u = x + 1$ (3 marks)

(c) $\int \sin^2 x \cos^3 x \; dx$ (3 marks)

Question 2 **(5 marks)**

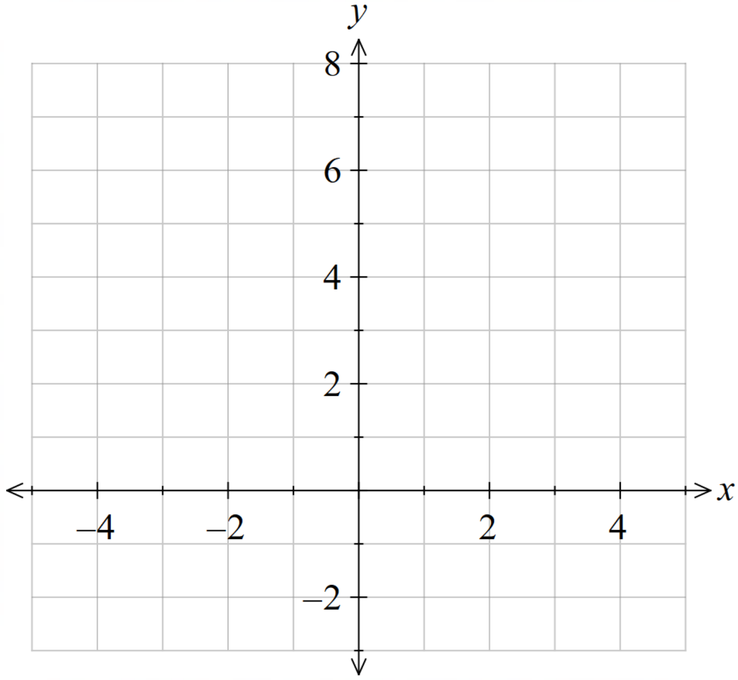
(a) Express $\dfrac{2x+1}{x^2(x+1)}$ in the form $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+1}$ (3 marks)

(b) Hence, determine $\displaystyle\int \frac{2x+1}{x^2(x+1)}\ dx$ (2 marks)

Question 3 **(6 marks)**

(a) On the axis below, sketch the slope field for the differential equation $\frac{dy}{dx} = 2y$. (2 marks)

(*Sketch only for even integer co-ordinates*)



(b) If $y(-1) = 1$, solve the differential given in part (a) to find y in terms of x. (3 marks)

(c) Sketch the graph of the solution curve found in part (b) on the slope field in part (a). (1 mark)

Question 4 **(5 marks)**

The curve $x^3 + y^3 - 9xy = 0$, known as a *folium*, dates back to Descartes in the 1630s.

(a) Determine $\dfrac{dy}{dx}$. (3 marks)

(b) Determine the equation of the tangent to the curve at the point (2, 4). (2 marks)

END OF QUESTIONS

TRINITY COLLEGE

IN NOMINE DOMINI

YEAR 12
MATHEMATICS
SPECIALIST

Test 3, 2023
Section Two: Calculator Allowed
Integration Techniques and Differential Equations

STUDENT'S NAME: ______________________________

DATE: Wednesday 16th August **TIME:** 28 minutes **MARKS**: 32
ASSESSMENT %: 10

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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Question 5 **(4 marks)**

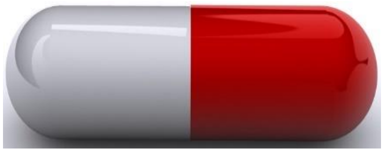
A refrigerator has a constant temperature of 3°C. A can of drink with temperature 30°C is placed in the refrigerator. After being in the refrigerator for 15 minutes, the temperature of the can of drink is 28°C. The change in the temperature of the can of drink can be modelled by $\frac{dT}{dt} = k(T-3)$, where T is the temperature of the can of drink, t is the time in minutes after the can is placed in the refrigerator and k is a constant.

(a) Show that $T = 3 + Ae^{kt}$, where A is a constant, satisfies $\frac{dT}{dt} = k(T-3)$. (1 mark)

(b) After 60 minutes, at what rate is the temperature of the can of drink changing? (3 marks)

Question 6 **(5 marks)**

A balloon is in the shape of a cylinder and has hemispherical ends of the same radius as that of the cylinder. (i.e., it looks like a medicine capsule). The balloon is being inflated at the rate of 261π cubic centimetres per minute. At the instant that the radius of the cylinder is 3 cm, the volume of the balloon is 144π cubic centimetres and the radius of the cylinder is increasing at the rate of 2 centimetres per minute.

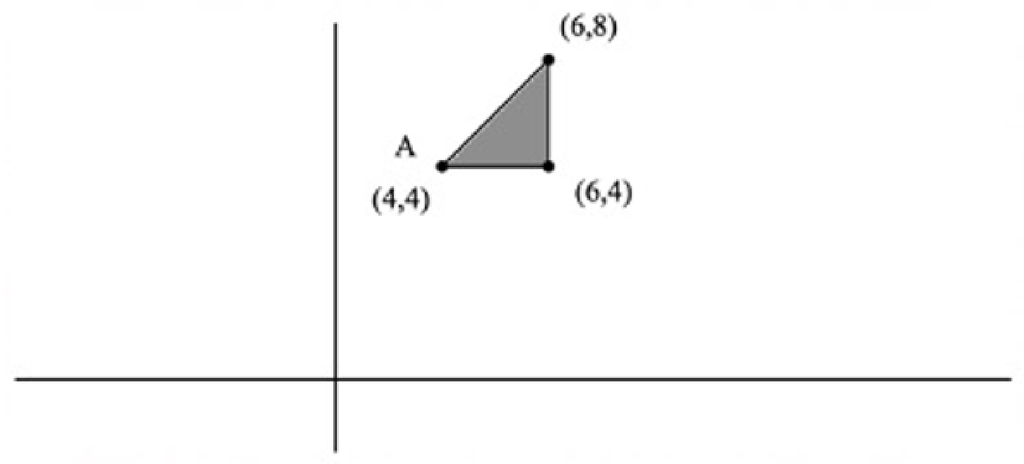


a) At this instant, what is the height of the cylinder? (2 marks)

b) At this instant, how fast is the height of the cylinder changing? (3 marks)

Question 7 **(10 marks)**

Consider the following shaded region.



(a) Determine the volume of the object created by rotating the shaded region around the x axis. (2 marks)

(b) Show that a horizontal translation of 1 unit to the right will not change the volume of the solid produced by rotating the shaded region around the x axis. (2 marks)

(c) State the vertical translation which would need to take place on the original shaded region which would give a volume of 500 units^2 when the shaded region is rotated around the x axis.
(3 marks)

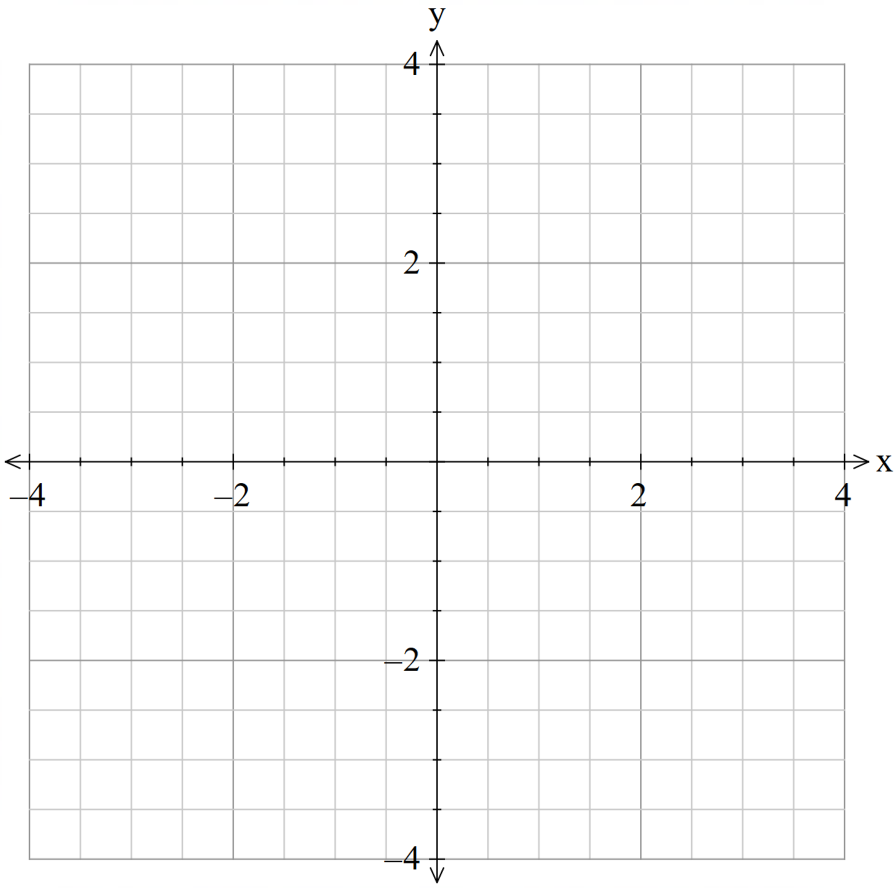
(d) Determine the volume of the object created by rotating the original shaded region around the y axis. How does this compare to your answer in part (a). (3 marks)

Question 8 **(13 marks)**

Consider the function $f(x) = \log_e(4 - x^2)$.

(a) Determine the largest possible domain for which $f(x)$ is defined. (1 mark)

(b) Sketch the graph of $f(x)$, labelling all the key features and using exact values. (3 marks)



Let A be the magnitude of the area enclosed by the graph of $f(x)$, the co-ordinate axes and the line $x = 1$.

(c) Without evaluating A, use the graph of $f(x)$ to explain why $\log_e(3) < A < \log_e(4)$. (2 marks)

(d) i) Show that $\frac{d(x\log_e(4-x^2))}{dx} = \ln(4-\ x^2) - 2(\frac{x^2}{4-x^2})$. (2 marks)

All steps of working must be shown.

ii) Using the fact that $\dfrac{x^2}{4-x^2}$ can be written as $\dfrac{x^2-4+4}{4-x^2}$, show that

$\int \dfrac{x^2}{4-x^2}dx\ = \ln|x+2| - \ \ln\lceil x-2\rceil - x + c$. (3 marks)

iii) Hence find the exact value of A in the form $a + b\ \log_e(c)$ where a, b and c are integers. (2 marks)

END OF QUESTIONS